Let (*x1, ..., xn*) ∈ (*0, ∞*)*n* be a realisation of independent on [*0, θ*] uniformly distributed random variables *X1, ..., Xn*. What is Maximum Spacing Estimator in this case? Using the data set provide on Moodle computer the unknown parameter *θ* via the Maximum spacing estimator for the three different sets of samples (note that they are of different sizes).

**Solution:**

Assume that *x (1), …., x(n)* are the ordered samples from a uniform distribution *U [0, θ]* with unknown endpoints *θ*. The cumulative distribution function is [1]:

………………………………………………...… (1)

Therefore, individual spacings are given by

………………………………………………...… (2)

…………………………………………. (3)

…………………………..……………………. (4)

When the geometric mean\* is calculated and then the logarithm is taken and then the *Sn* will be:

….... (5)

In equation (5) only the third term depends on the parameters . Differentiating with respect to , we got

*=*

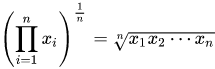
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*=*  ……………………………. (6)

\*The geometric mean is defined as the *nth* root of the product of n numbers, i.e., for a set of numbers *x1, x2, ..., xn*, the geometric mean is defined as [3]:



Solving the maximum spacing estimates estimators of is:

* …………………………………..……. (7)

Now, for “sampleset\_1\_problemsheet4\_ex1.txt”, if we sort all the *x* then so according to equation (7)

for “sampleset\_2\_problemsheet4\_ex1.txt”, if we sort all the *x* then so according to equation (7)

for “sampleset\_2\_problemsheet4\_ex1.txt”, if we sort all the *x* then so according to equation (7)

Reference:

1. Maximum spacing estimation, https://en.wikipedia.org/wiki/Maximum\_spacing\_estimation
2. Cheng, R. C. H., & Amin, N. A. K. (1983). Estimating parameters in continuous univariate distributions with a shifted origin. Journal of the Royal Statistical Society: Series B (Methodological), 45(3), 394-403.
3. Geometric mean, https://en.wikipedia.org/wiki/Geometric\_mean